LETTER TO THE EDITOR

Comments on the Dipole Model and Phase Transitions in Biological Membranes

Dear Sir:

I am presently involved in numerically estimating the dipole-dipole interaction energy in the case of the dipole model of an excitable membrane. In this connection, I read with interest a recent paper of Almeida et al. (1971) published in this journal, the central point of which is to estimate the transition temperature using the phase transition condition of a Lenz-Ising type model. With reference to their paper, I would like to make the following points.

For the case of nearest neighbor interaction in a plane (two dimensions), the model was exactly solved by Onsager (1944). A more readable treatment of this may be found in Huang (1965). In view of this, equation (2) of Almeida et al. (1971) should be replaced by

$$\frac{\omega}{kT_c} = -7.05.$$

Further, as a result of some mix-up of units, some changes must be made in the values of the relevant parameters in order to obtain the transition temperature in the right neighborhood. To illustrate this, we calculate below the transition temperature using the parameters listed by Almeida et al. (1971), namely, $| P_A | = 200$ debye, $| x_A - x_B | = 5$ A, and $K = 80$ in their expression 5 (which in rationalized units should contain the factor $4\pi K\epsilon$ in the denominator) gives

$$\omega = \frac{-2 | P_A |^2}{4\pi K\epsilon_0 | x_A - x_B |^3},$$

$$= \frac{8.99 \times 10^9}{80} \text{ N} - \text{m}^2/\text{coul}^2 \times \frac{(-2)(200 \text{ debye} \times 3.34 \times 10^{-30} \text{ coul} - \text{m/debye})^2}{(5 \text{ A} \times 10^{-10} \text{ m/A})^3},$$

$$\simeq -8 \times 10^{-19} \text{ J}.$$

Since the value of c, the number of nearest neighbor dipoles in Lenz-Ising model is 4, the above value of ω gives for the transition temperature T_c

$$T_c = \frac{-\omega}{k} = 5.8 \times 10^4 \text{ °K},$$

where k = Boltzmann's constant = 1.38×10^{-23} J/°K.

The values of the parameters, however, can be chosen so as to obtain the right transition temperature. In order for the value of transition temperature to calculate around 300°K, one must choose a lower value of the dipole moment than $| P_A | = 200$ debye or a larger value for the dipole separation than $| x_A - x_B | = 5$ A or a combination thereof. Using the exact formula given above, the combinations of dipole moment and dipole separation that give $T_c \simeq 300$°K are (20 debye, 5 A) or (200 debye, 25 A). It should be noted at this point that the above extremes of values instead of being inconsistent, actually point towards two differing viewpoints currently being held about the structure of an excitable membrane.

A value of $| P_A | = 20$ debye may seem unreasonable if compared with the range of dipole moments of proteins as quoted by Wei (1969), namely, 170–1400 debye units, it is by no means clear that such dipole moments should in fact be attributed to proteins in the membranes. If the dipoles are attributed to individual phospholipid heads, then a 5 A separation of charges in the phospholipid head would give rise to a dipole moment 24 debye, compatible with the first of the above pairs ofvalues. Values of both the parameters are likely to increase if more than one phospholipid molecule combine to give the dipole unit. In particular if one considers the suggestion made by Kilkson (1970) that perhaps the membranes exist in a discrete domain structure, then one would expect large dipole moments and large separation, consistent with the second of the above pair of values. We must, of course, await better experimental data before choosing between these opposing viewpoints.

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